

The theme of this month's Calendar is geoboards.

Both the horizontal and vertical distances between adjacent pegs on the square geoboard equal 1. Find the area of the region that is common to both triangles.



1

A line l intersects the positive x - and y -axes at $A(a, 0)$ and $B(0, b)$, respectively. Lines parallel to the x - and y -axes are drawn through a point Q on l , intersecting the x -axis at R and the y -axis at P . Rectangle $OPQR$ and triangle QAR have the same area. Find the coordinates of Q in terms of a and b . (Point O is the origin.)

2

Four pegs have been removed from each corner of a 5×5 geoboard (6 pegs on a side), leaving a board with 20 pegs, as shown. How many squares can be formed on this new geoboard?



3

Triangle ABC with coordinates $A(-2, -2)$, $B(-2, -7)$, and $C(-14, -7)$ is reflected in the line $x + y = k$. Let $\triangle A'B'C'$ be the image of $\triangle ABC$. If the sum of the x - and y -coordinates of A' , B' , and C' is 58, find the value of k .

4

Let $m > 0$. Find the value of c such that the area bounded by $y = mx$, $x = 3$, $x = 9$, and $y = 0$ is bisected by $x = c$.

5

A square with area 25 and a circle with radius 4 are centered at the origin. The square's sides are parallel to the x - and y -axes. How many lattice points (points with integral coordinates) lie outside the square and inside the circle?

6

The area of the largest square that can be made on a square geoboard with $n + 1$ pegs on a side is n^2 . (We assume that the vertical and horizontal distances between adjacent pegs equal 1.) For $n > 1$, what is the area of the largest square that does not use any of the geoboard's 4 corner pegs? Give your answer in terms of n .

7

Let the horizontal and vertical distances between adjacent pegs on a square geoboard equal 1. If the geoboard has 4 pegs per side, find 4 noncongruent trapezoids, each with area 1.5.

8

We are given a square geoboard with 4 pegs on a side. If we select 3 pegs at random, what is the probability that the 3 pegs are the vertices of a triangle?

9

Suppose that $f(x) = mx + b$ is a linear function with positive slope. If

$$f(f(x)) = 25x + 9,$$

find the y -intercept of $f(x)$.

10

Two attributes are used to classify all triangles: the number of congruent sides (scalene, isosceles, equilateral) and the measure of the largest angle (acute, right, obtuse). Consider all triangles that can be formed on a 9-peg square geoboard and those that can be formed on a 16-peg square geoboard. What types of triangles can be formed on the latter but not on the former?

11

Two pegs on a 2×2 geoboard are connected by a geoband, as shown. Two different pegs are selected at random and connected by another geoband. If the second band is parallel to the first, what is the probability that the 4 pegs are vertices of a parallelogram?

12

Let the horizontal and vertical distances between adjacent pegs on a square geoboard equal 1. If the geoboard has 4 pegs per side, find 3 noncongruent kites, each with area 3 units².

13

A circular geoboard consists of $2n$ pegs, $n \geq 2$, equally spaced on the circumference of a circle. How many noncongruent right triangles can be formed on this geoboard?

14

Give the factorization of

$$1 + x^2 + x^4$$

into two quadratic factors with real coefficients.

15

On the square geoboard shown below, the distance between adjacent horizontal and vertical pegs is 1 in. What is the area of the 16-gon?



16

The horizontal and vertical distances between adjacent pegs on the square-array geoboard shown here are equal. If the outlined figures have a total area of 17 units², how far apart are consecutive horizontal pegs?



17

How many integers satisfy the inequality

$$\frac{1}{3} < \frac{n}{12} < \frac{4}{5}?$$

18

The surface area of a cube is 450. Find the length of an interior diagonal of the cube.

19

Suppose that

$$f(x) = 1/(1 + x).$$

Determine all values of x that are *not* in the domain of

$$f(f(f(x))).$$

20

For the arithmetic sequence $1/2, 1, 3/2, \dots, 9/2$, is it possible to form triangles on a 3×3 geoboard (with 4 pegs on a side) that have an area equal to each term of this sequence?

21

If $(x + 2)$ and $(x - 3)$ are factors of the polynomial

$$p(x) = x^5 - 5x^4 - x^3 + 19x^2 + ax + b,$$

find

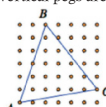
$$a + b.$$

22

Michael plays several rounds of a game in which he begins with n dollars. When he wins, he gains half of what he has; when he loses, he loses half of what he has. He wins on all the odd rounds and loses on all the even rounds. After the tenth round, he has \$121.50. Find n .

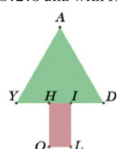
23

We cannot construct an equilateral triangle on a standard square geoboard, but we can approximate an equilateral triangle with the triangle shown. Horizontal and vertical pegs are 1 in. apart. By what percent does the area of $\triangle ABC$ differ from the area of a true equilateral triangle with side length AB ?



24

An equilateral triangle rests on a rectangular base as shown, with $YH:HI:ID = 3:2:3$ and with $HO = 2 \cdot HI$. The distance from A to \overline{OL} is 12 ft. Find the sum of the areas of rectangle $HOLI$ and triangle DAY .



25

Nick has three types of candy to pack in gift bags: chocolate truffles, peppermint sticks, and chocolate-covered almonds. Each bag must contain exactly 6 pieces and at least 1 piece of each type of candy. How many different gift bags can Nick create?

26

Find all values of x such that the mean of 77, 137, and x is 1 more than their median.

27

The diagonals of a rhombus have length 18 and 20. What is the perimeter of the rhombus?

28

The dimensions of a rectangular prism are consecutive integers, and its surface area is 2644 units². Find the volume of the prism.

29

How many noncongruent, nonsquare rectangles can be formed on a square geoboard with 5 pegs on a side?

30

A square has been partitioned into 4 rectangles, 2 of which are the squares shown in blue. The smaller blue square has side length 1. Using a straightedge and a compass, construct a single square whose area is equal to the sum of the areas of the 2 blue squares.



31